Majorana Zero Modes in a Kitaev Chain

Based on:

Alexei Yu. Kiteav, Phys.-Usp. 44, 131 (2001).

arXiv: cond-mat/0010440

Presented by: Ruihao Li

CWRU CMP Journal Club,

03/25/2022

Microsoft Strikes Again

Condensed Matter Theory Center @condensed_the · Mar 14
News Flash: **Microsoft** announces in a breakthrough team effort the likely observation of Majorana modes and topological gap, which would be the foundation for a fault-tolerant topological **quantum** computer, at a conference in Santa Barbara over the weekend

Tom Wong @thomasgwong · Mar 14
Topological quantum computing might be back in the running! @Microsoft claims evidence of Majorana zero modes. news.microsoft.com/innovation-sto…

For a refresher on its setback, see @philipcball's Sept 2021 article in @QuantaMagazine: quantamagazine.org/major-quantum-….

John Preskill @preskill · Mar 15
It's striking to hear Chetan Nayak's confident tone in this conversation with Sankar Das Sarma about recent progress by **Microsoft**. If the evidence holds up for a robust **topological** phase in a **quantum** wire, the next step is a topologically protected qubit.

Stephan Roche @StephanSroche · Mar 15
In a historic milestone, Azure Quantum demonstrates formerly elusive physics needed to build scalable topological qubits

Frank Wilczek @FrankWilczek · Mar 17
. @microsoft announces convincing Majorana mode, a big step toward **topological** qubits and powerful **quantum** computers bit.ly/3qbmdOw .
Extended discussion and explanation here:

Sergey Frolov @spinespresso · Mar 14
I agree it is easier to prove Majorana without showing any data than when you show some data.

Microsoft Strikes Again

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Microsoft Research Blog

Microsoft has demonstrated the underlying physics required to create a new kind of qubit

Published March 14, 2022

By Dr. Chetan Nayak, Distinguished Engineer

Research Area

Quantum computing

In a historic milestone, Azure Quantum demonstrates formerly elusive physics needed to build scalable topological qubits

Jennifer Langston

Mar 14, 2022

Microsoft's Azure Quantum program has developed devices that can create quantum properties which scientists have imagined for nearly a century but have not been able to unambiguously produce in the real world — until now.

It's a key scientific breakthrough that demonstrates the elusive building blocks for a topological quantum bit, or qubit, which Microsoft has long pursued as the most promising path to developing a scalable quantum computer that will launch a new generation of as-yet-unimagined computing capabilities for Azure customers.

Some Backgrounds

An interesting and yet controversial history of Majorana zero modes (MZMs) and topological quantum computation (TQC):

- 1980–1997: active exploration of anyons & topological phases/orders
- 1997–2007: idea of TQC introduced & explored (led by Kitaev et al.)
- 2007–2012: practical proposals for realizing MZMs discovered (Fu & Kane; Maryland & Weizmann groups)
- 2012–now: active experimental search for Majoranas

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. MOURIK, K. ZUO, S. M. FROLOV, S. R. PLISSARD, E. P. A. M. BAKKERS, AND ,

SCIENCE • 12 Apr 2012 • Vol 336, Issue 6084 • pp. 1003-1007 • DOI: 1

Published: 09 March 2016

Exponential protection of zero modes in Majorana islands

S. M. Albrecht, A. P. Higginbotham, M. Madsen, F. Kuemmeth, T. S. Jespersen, J. Nygård, P. Krogstrup &

cs

> "I don't know for sure what was in their heads… but they skipped some data that contradicts directly what was in the paper. From the fuller data, there's no doubt that there's no Majorana."
> — Remark by Sergey Frolov

Majorana bound state in hybrid-nanowire system

M. T. DENG, S. VAITIEKĖNAS, E. B. HANSEN, J. DANON, M. LEIJNSE, K. FLENSBERG, J. NYGÅRD, P. KROGSTRUP, AND C. M. MARCUS Authors Info & Affiliations

SCIENCE • 23 Dec 2016 • Vol 354, Issue 6319 • pp. 1557-1562 • DOI: 10.1126/science.aaf3961

ized Majorana

, John A. Logan, Guanzhong Wang, Nick van Loo, Jouri D. S. Bommer, Michiel W. A. de Moor, Diana Car, Roy L. M. Op het Veld, Petrus J. van Veldhoven, Sebastian Koelling, Marcel A. Verheijen, Mihir Pendharkar, Daniel J. Pennachio, Borzoyeh Shojaei, Joon Sue Lee, Chris J. Palmstrøm, Erik P. A. M. Bakkers, S. Das Sarma & Leo P. Kouwenhoven

Nature **556**, 74–79 (2018) | Cite this article

44k Accesses | **407** Citations | **318** Altmetric | Metrics

This article was retracted on 08 March 2021

False positives — suffer from "confirmation bias": disorder, Andreev bound states, etc.

Topological Quantum Computation

- Challenge in building a scalable quantum computer: quantum decoherence (environmental noises, charge fluctuations, etc.)
  - Relaxation: perturbation perpendicular to quantization axis — bit flips
  - Dephasing: perturbation along quantization axis

- Active error corrections: repetition code (bit flip), Shor's 9-qubit code (arbitrary single-qubit errors), surface code (better scalability)...

  Expense: large overhead of qubits

- Topological quantum computation (Kitaev):

  Quantum information encoded in topological (necessarily nonlocal) degrees of freedom, which are insensitive to local probes.

  $\Longrightarrow$ Fault tolerant at the hardware level

More on Kitaev's Argument

- How to suppress bit-flip errors?
  - Encode $|0\rangle$ and $|1\rangle$ states with empty and occupied electron sites
  - Single bit-flip errors are impossible b/c. fermionic parity conservation
  - Electron jumps avoided by placing the fermionic sites far apart

- How about dephasing errors?
  - Dephasing described by operators $a_j^\dagger a_j$
  - One may define Majorana operators

$$\gamma_{2j-1} = a_j + a_j^\dagger, \qquad \gamma_{2j} = -i(a_j - a_j^\dagger) \qquad (j = 1, \ldots, N)$$

$$\gamma_i^\dagger = \gamma_i, \qquad \{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

$$a_j^\dagger a_j = \frac{1}{2}(1 + i\gamma_{2j-1}\gamma_{2j})$$

  - An isolated Majorana site is immune to error (again due to fermionic parity conversation)

The Kitaev Chain

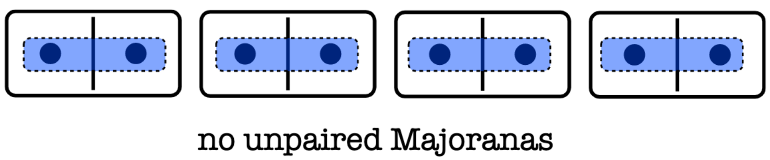
- 1D chain of spinless fermions:

$$H = -\mu \underbrace{\sum_{n=1}^{N} (a_n^\dagger a_n - \frac{1}{2})}_{\text{Onsite energy}} \underbrace{-t \sum_{n=1}^{N} (a_{n+1}^\dagger a_n + \text{h.c.})}_{\text{Hopping}} \underbrace{+\Delta \sum_{n=1}^{N} (a_n a_{n+1} + \text{h.c.})}_{\text{Superconducting pairing}}$$

- Trivial limit:

$|\Delta| = t = 0,\ \mu < 0:$

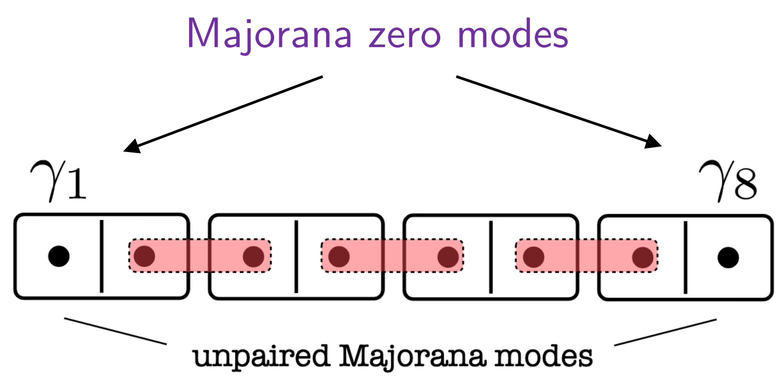
$$H = -\mu \sum_{n=1}^{N} (a_n^\dagger a_n - \frac{1}{2}) = -\frac{i}{2}\mu \sum_{n=1}^{N} \gamma_{2n-1}\gamma_{2n}$$



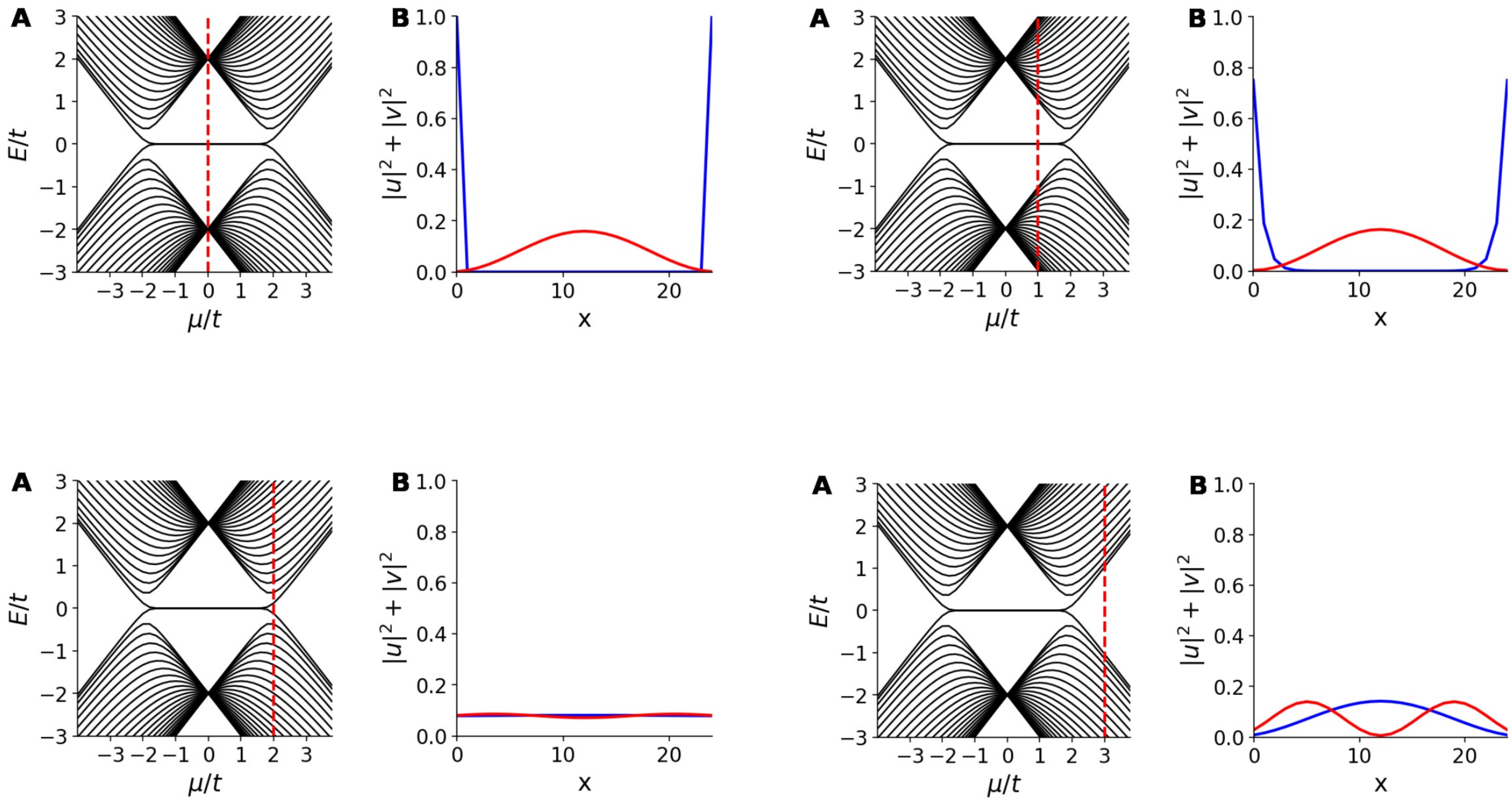
- "Topological" limit:

$|\Delta| = t > 0,\ \mu = 0:$

$$H = it \sum_{n=1}^{N-1} \gamma_{2n}\gamma_{2n+1}$$



Finite Kitaev Chain



$$-2t \leq \mu \leq 2t: \ \text{topological phase}$$

Topological Phase from Bulk Spectrum

- Bogoliubov-de Gennes formalism:

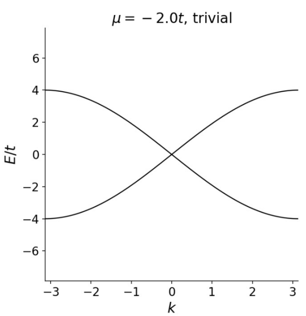
$$H = \frac{1}{2} C^\dagger H_{\mathrm{BdG}} C$$
$$C = (c_1, \ldots, c_N, c_1^\dagger, \ldots, c_N^\dagger)^T$$
$$\Longrightarrow \quad H_{\mathrm{BdG}} = -\sum_n \mu \tau_z \left| n \right\rangle \left\langle n \right| - \sum_n \left[(t\tau_z + i\Delta\tau_y) \left| n \right\rangle \left\langle n+1 \right| + \mathrm{h.c.} \right]$$

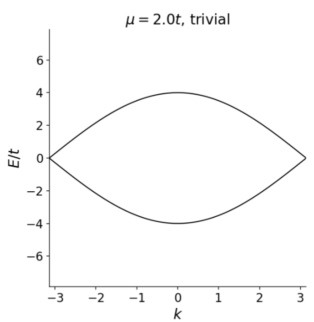
Particle-hole symmetry: $\mathcal{P} H_{\mathrm{BdG}} \mathcal{P}^{-1} = -H_{\mathrm{BdG}}, \text{ with } P = \tau_x \mathcal{K}$

- "Kitaev ring": periodic boundary condition

$$\left| n \right\rangle = \frac{1}{\sqrt{N}} \sum_k e^{ikn} \left| k \right\rangle$$
$$\Longrightarrow \quad H_{\mathrm{BdG}} = \sum_k \underbrace{\left[(-\mu - 2t\cos k)\tau_z + 2\Delta \sin k \, \tau_y \right]}_{\equiv H(k)} \left| k \right\rangle \left\langle k \right|$$

$$E(k) = \pm\sqrt{(\mu + 2t\cos k)^2 + 4\Delta^2 \sin^2 k}$$





Bulk Topological Invariant

- Insight: Gap closings correspond to fermionic parity switches
  - The excitation energy of the Bogoliubov quasiparticle changes sign as a pair of levels crosses zero energy
  - It becomes favorable to add/remove a Bogoliubov quasiparticle
  - This changes the fermionic parity of the ground state

- Pfaffian (for 2n x 2n antisymmetric matrix A):

$$\mathrm{Pf}(A) = \frac{1}{2^n n!} \sum_{\sigma \in S_{2n}} (-1)^{|\sigma|} \prod_{i=1}^{n} A_{\sigma(2i-1),\sigma(2i)}$$

- Basis transformation: $U = e^{-i\pi\sigma_y/4} = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$

$$\mathrm{Pf}\left[i\tilde{H}(0)\right] = \mathrm{Pf}\left[\frac{1}{2} iUH(0)U^\dagger\right] = -2t - \mu$$

$$\mathrm{Pf}\left[i\tilde{H}(\pi)\right] = \mathrm{Pf}\left[\frac{1}{2} iUH(\pi)U^\dagger\right] = 2t - \mu$$

Bulk Topological Invariant

- Can define a bulk invariant:

$$\mathcal{M} = \mathrm{sign}\Big(\mathrm{Pf}\big[i\tilde{H}(0)\big]\,\mathrm{Pf}\big[i\tilde{H}(\pi)\big]\Big) = \begin{cases} -1 & \text{topological phase} \\ +1 & \text{trivial phase} \end{cases}$$

- Alternative invariant when $\Delta \in \mathbb{R}$: winding number (cf. SSH model)

- Topological classification: Class D or BDI (when $\Delta \in \mathbb{R}$)

Chiu, Teo, Schnyder, Ryu, RMP **88**, 035005 (2016)

Altland-Zirnbauer Random Matrix Classes

	Symmetry			Spatial Dimension d								
Class	T	C	S	1	2	3	4	5	6	7	8	⋯
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	⋯
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	⋯
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	⋯
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	⋯
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	⋯
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	⋯
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	⋯
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	⋯
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	⋯
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	⋯

time-reversal : $TH(k)T^{-1} = H(-k); \quad T^2 = \pm 1$

particle-hole : $CH(k)C^{-1} = -H(-k); \quad C^2 = \pm 1$

sublattice/chiral $SH(k)S^{-1} = -H(k); \quad S \propto TC$

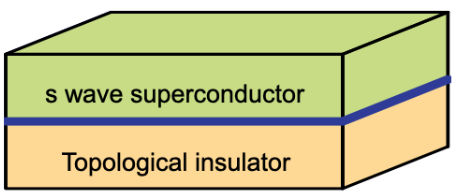
Physical Realizations

Intrinsic p-wave superconductors are very rare in nature ($\nu = 5/2$-FQH; Sr_2RuO_4?)

Basic ingredients to realize a 1D spinless p-wave superconductor (Kiteav chain) in engineered platforms:

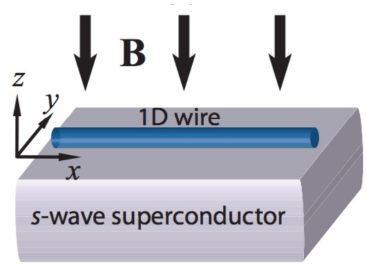
- Proximity coupling to a conventional s-wave superconductor
- Spin polarization
- Spin-orbit coupling

TI/SC



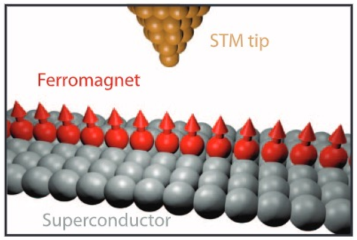
Fu & Kane, PRL **100**, 096407 (2008)
Fu & Kane, PRB **79**, 161408 (2009)

SM/SC



Lutchyn, Sau, Das Sarma, PRL **105**, 077001 (2010)
Oreg, Refael, von Oppen, PRL **105**, 177002 (2010)

FM atomic chain/SC



Nadj-Perge et al., PRB **88**, 020407(R) (2013)
Nadj-Perge et al., Science **346**, 602 (2014)

Summary

- History of Majorana zero modes
- Why Majorana fermions might be useful in topological quantum computation
- Kitaev Chain: 1D p-wave topological superconductor
  - Finite chain: conditions for topological phase
  - Bulk-edge correspondence (the Kitaev ring)
  - Bulk topological invariant and topological classification
- Synthetic platforms for realizing the Kitaev chain