Quantum Corrections in Left-Right Symmetric Seesaw Mechanisms

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THE UNIVERSITY OF SYDNEY

CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

Outline

Neutrinos

- Weakly interacting particles.
- Neutrino oscillations[1] indicate that they have non-zero masses:

$$P(\nu_\alpha \to \nu_\beta) \approx \sin^2(2\theta) \sin^2 \frac{\Delta m_{12}^2}{4E} L.$$

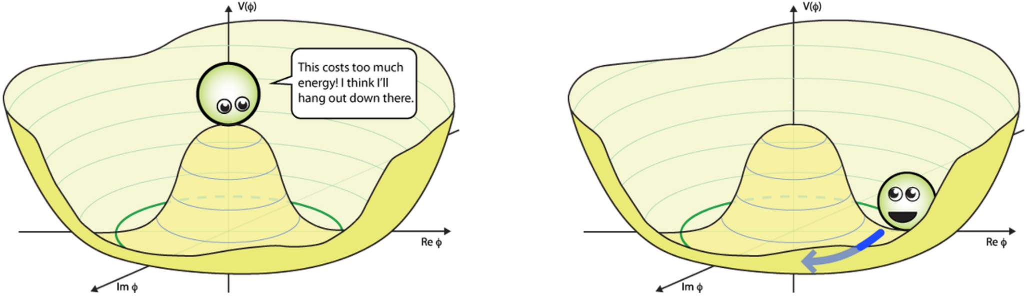
- Active neutrinos have tiny masses.
  - Effective neutrino mass bounds from $0\nu 2\beta$ decay[2]: $m_{ee} \lesssim 0.16$ eV
  - Neutrino mass bounds from cosmological probes[3]: $m_\nu = \sum_i m_i \lesssim 0.12$ eV
- Neutrino mass is not explained by the Standard Model (SM).

[1] Y. Fukuda *et al.* (Super-Kamiokande Collaboration), *Phys. Rev. Lett.*, **81**, 1562 (1998).
[2] A. Gando *et al.* (KamLAND-Zen Collaboration), Phys. Rev. Lett., **117**, 082503 (2016).
[3] N. Palanque-Delabrouille *et al.*, *JCAP*, **11**, 011 (2015).

Fermion Mass Terms

- A fermion (spinor) field is generally composed of the left-handed (L) and right-handed (R) chiral components.
- Fermions in the SM except neutrinos obtain their masses via the **Higgs mechanism**, e.g.

$$\mathcal{L}_e = -y_e(\overline{\ell_L}\phi e_R + \phi^\dagger \overline{e_R}\ell_L) \xrightarrow{\langle\phi\rangle=\begin{pmatrix}0\\v\end{pmatrix}} -y_e v(\overline{e_L}e_R + \overline{e_R}e_L)$$



Source: Quantum Diaries blog.

Dirac or Majorana?

- If ψ_L is completely independent of ψ_R, we have a **Dirac field**:

$$\mathscr{L}_D = -m_D(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L).$$

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- If ψ_L is the particle-antiparticle conjugation[4] of ψ_R,

$$\psi_L = \psi_R^{\mathcal{C}} \equiv \left(\psi^{\mathcal{C}}\right)_L,$$

  we have a **Majorana field**:

$$\mathscr{L}_M = -\frac{1}{2} m_R(\overline{\psi_R^{\mathcal{C}}}\psi_R + \overline{\psi_R}\psi_R^{\mathcal{C}}).$$

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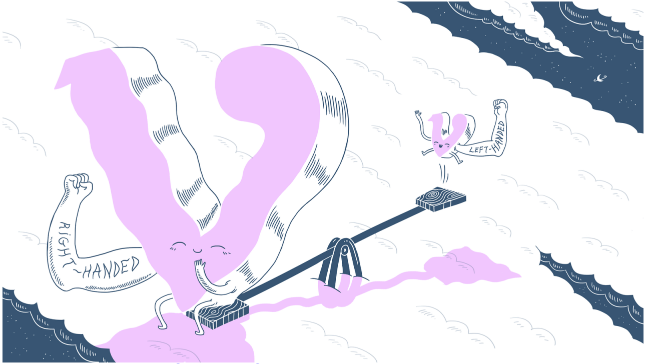
  $\Longrightarrow$ A tiny Yukawa coupling: fine-tuning problem
- If Neutrinos are Majorana fermions, they allow for a Dirac mass as well as a Majorana mass.
  $\Longrightarrow$ **Seesaw mechanism**



Source: Symmetry Magazine

Seesaw Mechanism

- If we introduce the **heavy RH neutrinos**:

$$\mathscr{L}_m = \boxed{-\frac{1}{2}\overline{N_R^{\mathcal{C}}} M_N N_R - \frac{1}{2}\overline{N_R} M_N N_R^{\mathcal{C}}} \; \boxed{-\overline{N_R}\, y_N \widetilde{\phi}^\dagger \ell_L - \overline{\ell_L}\widetilde{\phi}\, y_N^\dagger N_R}$$

Majorana mass terms

Dirac Yukawa coupling

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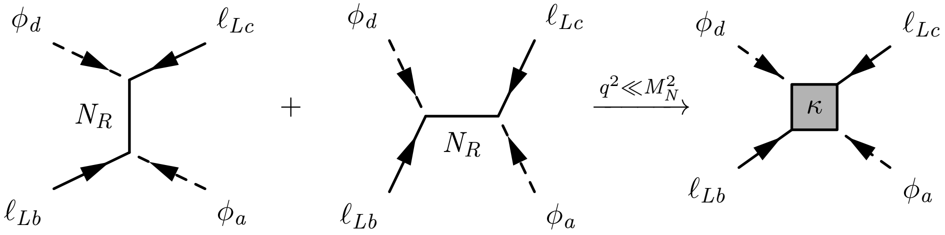
Majorana mass terms — Dirac Yukawa coupling

- Effective field theory:



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- After electroweak symmetry breaking:

$$\langle \phi_0 \rangle = v \quad \Longrightarrow \quad \mathscr{L}_\kappa = \frac{1}{2} \left(\frac{v^2}{2} \kappa \right) \overline{\nu_L^{\mathcal{C}}} \nu_L + h.c.$$

$$\Longrightarrow \quad m_\nu = -\frac{v^2}{2} \kappa = -v^2 y_N^T M_N^{-1} y_N$$

- We obtain the **type-I seesaw relation**:

$$\boxed{m_\nu = -v^2 y_N^T M_N^{-1} y_N.}$$

Seesaw Mechanism

- Instead of RH neutrinos, we can also introduce a **charged scalar triplet** $\Delta \sim (3,1)$:

$$\Delta = \frac{\sigma^i \Delta^i}{\sqrt{2}} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}.$$

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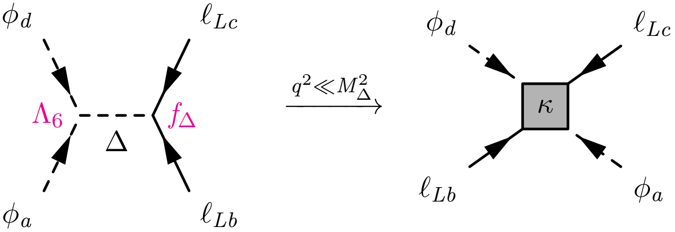

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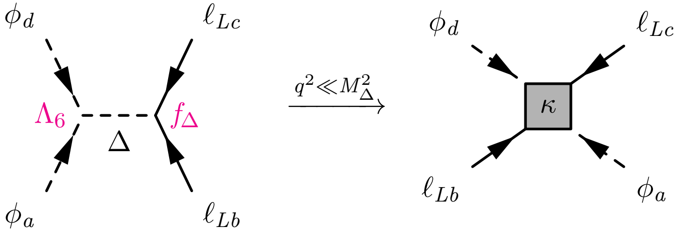

- Similar derivation yields the **type-II seesaw relation**:

$$m_\nu = v_\Delta f_\Delta \sim \left(\frac{\Lambda_6 v^2}{M_\Delta^2}\right) f_\Delta.$$

Outline

Gauge LR Symmetry

- The Standard Model gauge group:

$$G_{SM} = \underbrace{SU(3)_C}_{\text{color charge}} \times \underbrace{SU(2)_L}_{\text{weak isospin}} \times \underbrace{U(1)_Y}_{\text{hypercharge}} .$$

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- In the **left-right symmetric model (LRSM)**, the gauge group is extended:

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- Fermion sector:

$$Q_{L,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L,R}, \qquad \ell_{L,R} = \begin{pmatrix} \nu \\ e \end{pmatrix}_{L,R}$$

- Higgs sector:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}, \qquad \Delta_{L,R} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}_{L,R}$$

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  - $\langle\Delta_R\rangle \implies M_N = v_R f_R$
  - $\langle\Phi\rangle \implies m_\nu^I \simeq -\frac{v^2}{v_R} y^T f_R^{-1} y$ (type-I seesaw)
  - $\langle\Delta_L\rangle \implies m_\nu^{II} = v_L f_L$ (type-II seesaw)

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  - $\langle\Delta_L\rangle \Longrightarrow m_\nu^{II} = v_L f_L$ (type-II seesaw)
- The **LR symmetric seesaw relation** is

$$\boxed{m_\nu = m_\nu^{II} + m_\nu^I \simeq v_L f_L - \frac{v^2}{v_R} y^T f_R^{-1} y}$$

Discrete LR Symmetry

- The discrete charge conjugation symmetry ($\ell_L \leftrightarrow \ell_R^{\mathcal{C}}$) leads to

$$f_L = f_R \equiv f, \qquad\qquad y = y^T,$$

so that the LR symmetric seesaw relation

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can be solved analytically[5].

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- The parameters in the seesaw relation receive quantum corrections as they evolve towards lower energies ("**running**"), which will lead to the the violation of the discrete LR symmetry.
- Quantum corrections are described by the **renormalization group equations (RGEs)**.

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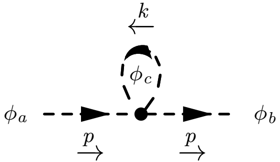
Outline

Taming Divergences

- Infinities show up when we calculate loop diagrams. For example,

$$i\Sigma_\phi =$$



$$= \frac{3}{2}\lambda \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2 + i\varepsilon} \to \infty.$$

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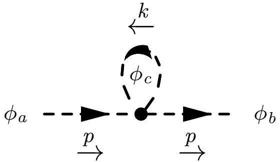

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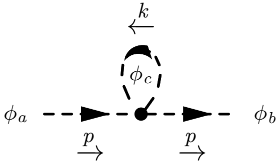

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- We use **dimensional regularization** to evaluate the integral.
- Key observation: such an integral is divergent in $d = 4$ but convergent in $d < 4$.

$$\begin{aligned} i\Sigma_\phi &= \frac{3}{2}\mu^\epsilon \lambda \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2 - m^2 + i\varepsilon} \\ &= i\frac{3}{32\pi^2}\lambda m^2\left[\frac{2}{\epsilon} + \ln\frac{4\pi\mu^2}{m^2} - \gamma_E + 1 + \mathscr{O}(\epsilon)\right], \end{aligned}$$

where $d = 4 - \epsilon$ and μ is some arbitrary mass scale introduced to keep the coupling constant dimensionless.

Counterterms

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- "Bare" quantities are not observable in experiments and so they can be divergent; the renormalized ones need to be finite!
- Define the relations between them

$$
\begin{aligned}
\phi_B &= {Z_\phi}^{1/2}\phi \\
m_B^2 &= {Z_\phi}^{-1}(m^2 + \delta m^2) \\
\lambda_B &= \mu^\epsilon {Z_\phi}^{-2} Z_\lambda \lambda.
\end{aligned}
$$

- To one-loop order, expand the renormalization constant:

$$
\begin{aligned}
Z_\phi &= 1 + \delta Z_\phi \\
Z_\lambda &= 1 + \delta Z_\lambda.
\end{aligned}
$$

$\delta Z_\phi,\ \delta Z_\lambda$ and δm^2 are called the **counterterms** used to cancel the divergences.

Counterterms

- Lagrangian of the Higgs potential in the SM:

$$\mathscr{L}_\phi = (\partial_\mu \phi_B)^\dagger (\partial^\mu \phi_B) - m_B^2 \phi_B^\dagger \phi_B - \frac{1}{4}\lambda_B (\phi_B^\dagger \phi_B)^2$$
$$= (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - m^2 \phi^\dagger \phi - \frac{1}{4}\mu^\epsilon \lambda (\phi^\dagger \phi)^2 +$$
$$\boxed{\delta Z_\phi (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \delta m^2 \phi^\dagger \phi - \frac{1}{4}\mu^\epsilon \delta Z_\lambda \lambda (\phi^\dagger \phi)^2}.$$

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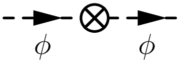
Counterterm Lagrangian

- The counterterm Lagrangian comes with the Feynman rule:

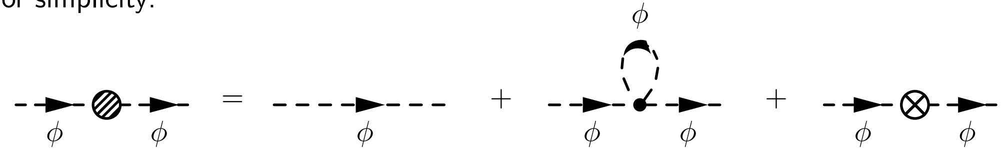


$$= i(p^2 \delta Z_\phi - \delta m^2).$$

MS Scheme

$$i\Sigma_\phi = i\frac{3}{32\pi^2}\lambda m^2\left[\frac{2}{\epsilon} + \ln\frac{4\pi\mu^2}{m^2} - \gamma_E + 1 + \mathscr{O}(\epsilon)\right]$$

- For simplicity:



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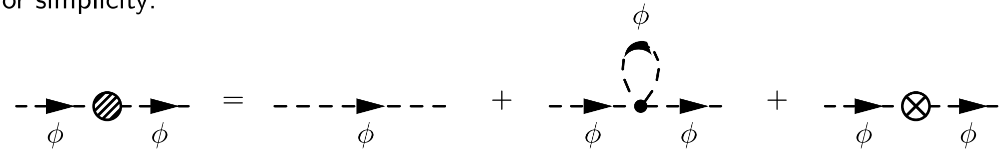

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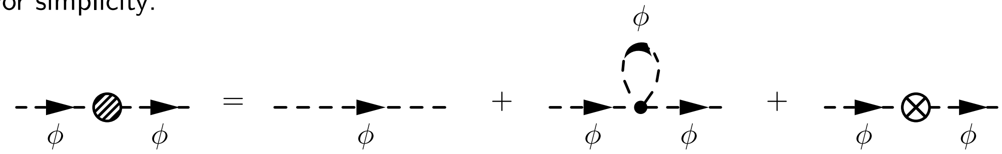

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- The **Minimal Subtraction (MS) scheme**: only the divergent part in the vertex function is absorbed in the counterterms.
- $$p^2\delta Z_\phi - \delta m^2 + \frac{3}{16\pi^2}\lambda m^2\frac{1}{\epsilon} = 0,$$

  which gives

$$\delta Z_\phi = 0$$

$$\delta m^2 = \frac{3}{16\pi^2}\lambda m^2\frac{1}{\epsilon}.$$

Renormalization Group Equations

- The renormalized quantities (e.g. m, λ) depend on the renormalization scale μ, but the bare quantities do not.

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- Consider the n-point Green function in momentum space with wavefunction renormalization $\phi_B = Z_\phi^{1/2}\phi$:

$$G_B^{(n)}(\{p_i\}, \lambda_B, m_B) = Z_\phi^{n/2} G^{(n)}(\{p_i\}, \lambda, m, \mu),$$

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- The bare Green function is independent of the renormalization scale μ:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} G_B^{(n)}(\{p_i\}, \lambda_B, m_B) = 0$$

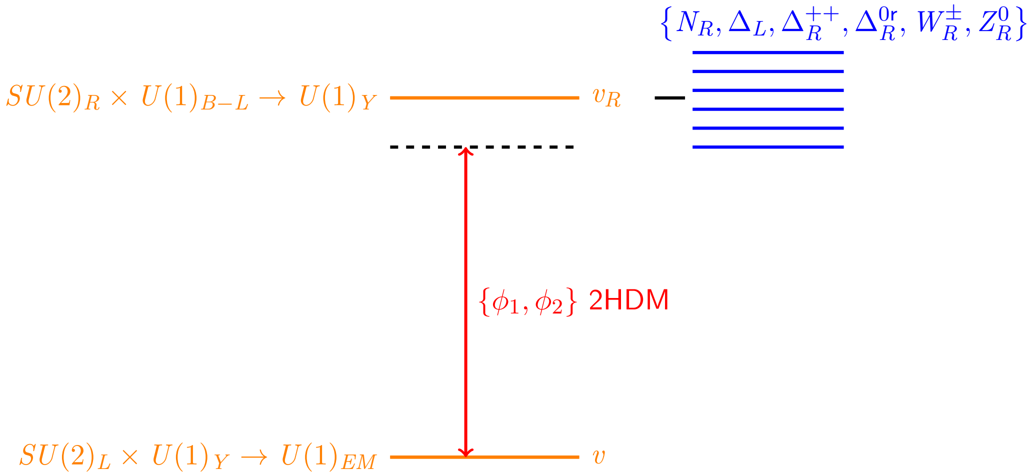
$$\Longrightarrow \boxed{\left[\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial\lambda} - \gamma_m m\frac{\partial}{\partial m} + \frac{n}{2}\gamma\right] G^{(n)}(\{p_i\}, \lambda, m, \mu) = 0}$$ Callan-Symanzik equation

where

$$\boxed{\beta \equiv \mu\frac{\mathrm{d}\lambda}{\mathrm{d}\mu}, \quad \gamma_m \equiv -\frac{1}{m}\mu\frac{\mathrm{d}m}{\mathrm{d}\mu}, \quad \gamma \equiv \frac{1}{Z_\phi}\mu\frac{\mathrm{d}Z_\phi}{\mathrm{d}\mu}}$$ RGEs

Outline

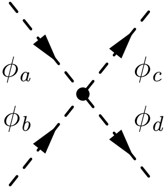
Relevant Particles



Aim: derive the RGE for the effective coupling κ in the **Two Higgs Doublet Model (2HDM)**.

Revisiting the Feynman Rules

- Higgs self-interaction in SM:



$$: \quad -i\mu^{\epsilon}\lambda\frac{1}{2}(\delta_{ca}\delta_{db}+\delta_{cb}\delta_{da})$$

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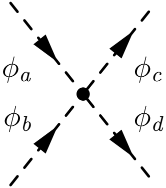
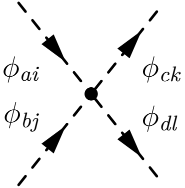


$$: \quad -i\mu^{\epsilon}\lambda\frac{1}{2}(\delta_{ca}\delta_{db}+\delta_{cb}\delta_{da})$$

- Higgs self-interaction in 2HDM (or n Higgs in general):

$$\mathscr{L}_{\phi} = \frac{1}{4}\lambda_{ijkl}\left(\phi_i^{\dagger}\phi_j\right)\left(\phi_k^{\dagger}\phi_l\right) \quad \Longrightarrow \quad \lambda_{ijkl} = \lambda_{klij}$$



$$: \quad -i\mu^{\epsilon}\frac{1}{2}\left(\lambda_{kilj}\delta_{ca}\delta_{db}+\lambda_{kjli}\delta_{cb}\delta_{da}\right)$$

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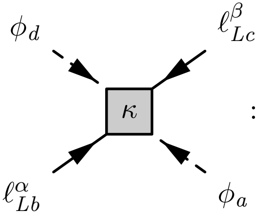

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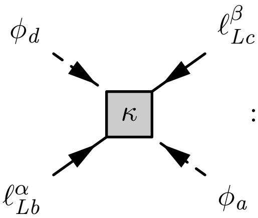
Revisiting the Feynman Rules

- Effective Lagrangian in SM: $\mathscr{L}_\kappa = \frac{1}{4}\kappa_{\beta\alpha}(\overline{\ell_L^{\mathcal{C}}}^{\beta}\,\varepsilon\phi)\;(\ell_L^{\alpha}\varepsilon\phi) + h.c.$



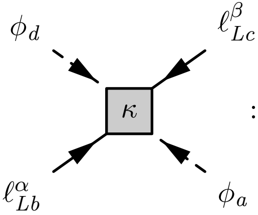
$$: \quad i\mu^{\epsilon}\kappa_{\beta\alpha}\frac{1}{2}(\varepsilon_{cd}\varepsilon_{ba} + \varepsilon_{ca}\varepsilon_{bd})$$

- Effective Lagrangian in 2HDM:

$$\mathscr{L}_\kappa = \frac{1}{4}(\kappa_1)^{ji}_{\beta\alpha}(\overline{\ell_L^{\mathcal{C}}}^{\beta}\,\varepsilon\phi_j)(\ell_L^{\alpha}\varepsilon\phi_i) + \frac{1}{4}(\kappa_2)^{ji}_{\beta\alpha}(\overline{\ell_L^{\mathcal{C}}}^{\beta}\,\varepsilon\ell_L^{\alpha})(\phi_j\varepsilon\phi_i) + h.c.$$

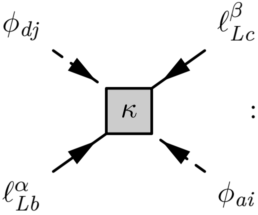
Revisiting the Feynman Rules

- Effective vertex in SM:



$$: \quad i\mu^{\epsilon}\kappa_{\beta\alpha}\frac{1}{2}(\varepsilon_{cd}\varepsilon_{ba}+\varepsilon_{ca}\varepsilon_{bd})$$

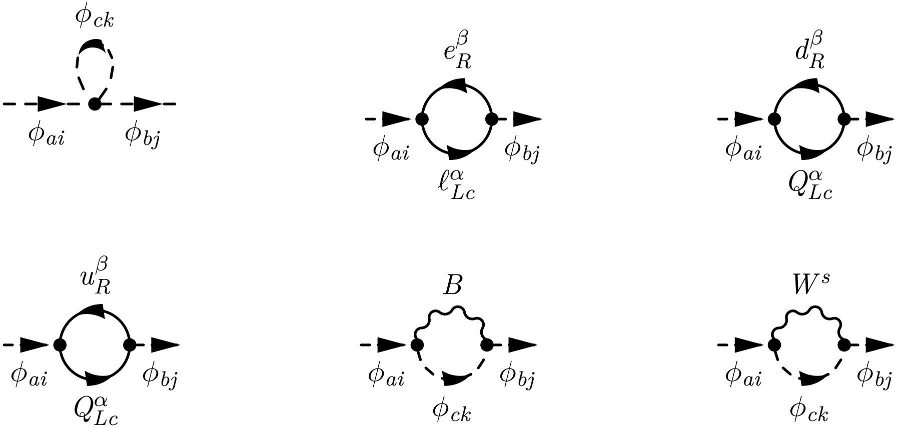
- Effective vertex in 2HDM:



$$: \quad i\mu^{\epsilon}\frac{1}{2}\left(\kappa^{ji}_{\beta\alpha}\varepsilon_{cd}\varepsilon_{ba}+\kappa^{ij}_{\beta\alpha}\varepsilon_{ca}\varepsilon_{bd}\right)$$

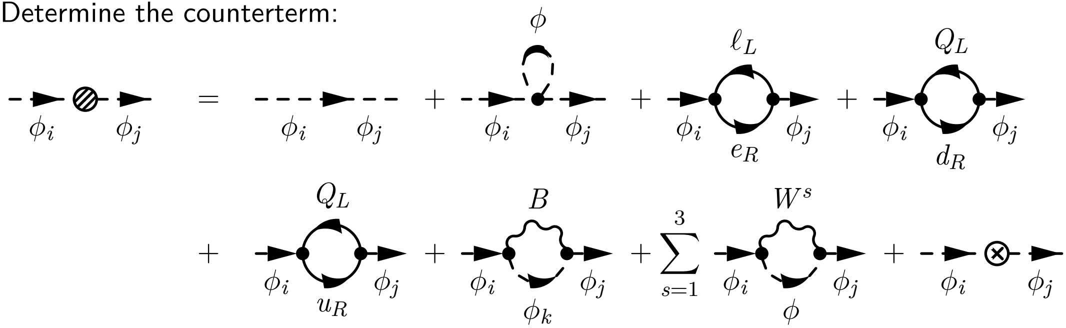
Higgs Wavefunction Renormalization

Relevant diagrams:



Higgs Wavefunction Renormalization

Determine the counterterm:



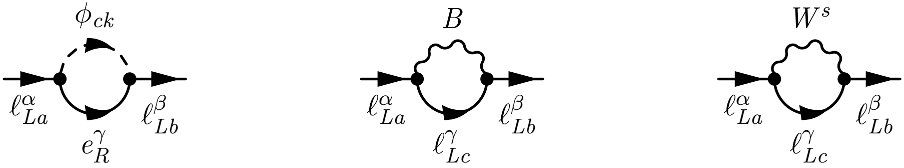
$$= \quad i\delta_{ij} + i \sum \text{all vertex corrections} + i\left[p^2 (\delta Z_\phi)^{ij} - \delta m^2_{ij}\right]$$

$$\stackrel{!}{=} \quad \text{finite.}$$

$$\Longrightarrow \quad (\delta Z_\phi)^{ij} = -\frac{1}{16\pi^2}\left[2\,\mathrm{Tr}\left(Y_e^{i\dagger}\,Y_e^j + 3\,Y_d^{i\dagger}\,Y_d^j + 3\,Y_u^{i\dagger}\,Y_u^j\right) - \frac{1}{2}g_1^2(3-\xi_1)\delta_{ij} - \frac{3}{2}g_2^2(3-\xi_2)\delta_{ij}\right]\frac{1}{\epsilon}.$$

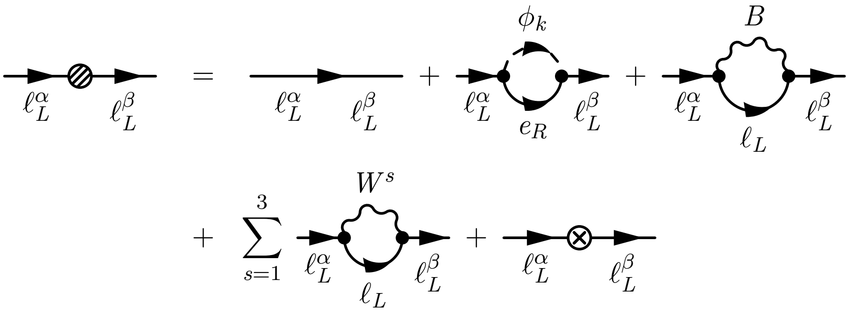
Lepton Doublet Wavefunction Renormalization

Relevant diagrams:



Lepton Doublet Wavefunction Renormalization

Determine the counterterm:



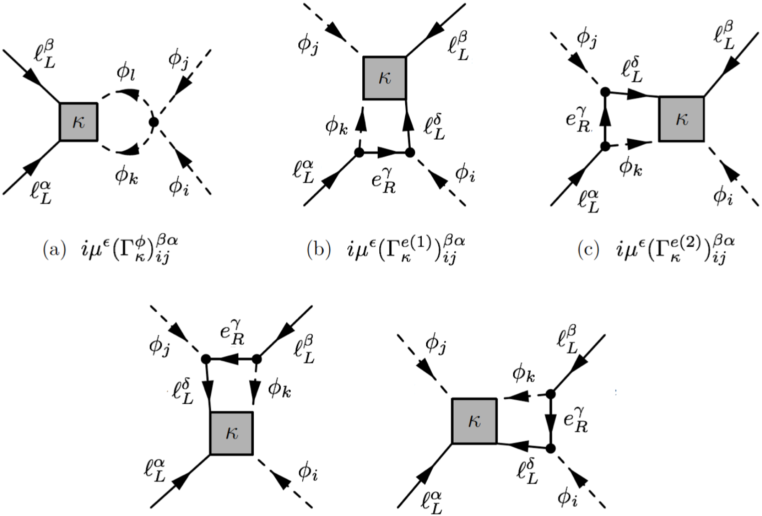
$$= \quad i\delta_{\beta\alpha} + i\sum \text{all vertex corrections} + i\not{p}(\delta Z_{\ell_L})_{\beta\alpha} P_L$$

$$\stackrel{!}{=} \quad \text{finite.}$$

$$\Longrightarrow \qquad (\delta Z_{\ell_L})_{\beta\alpha} = -\frac{1}{16\pi^2}\left[\sum_{k=1}^{2}\left(Y_e^{k\dagger}Y_e^{k}\right)_{\beta\alpha} + \frac{1}{2}\xi_1 g_1^2\delta_{\beta\alpha} + \frac{3}{2}\xi_2 g_2^2 \delta_{\beta\alpha}\right]\frac{1}{\epsilon}.$$

Effective Vertex Renormalization

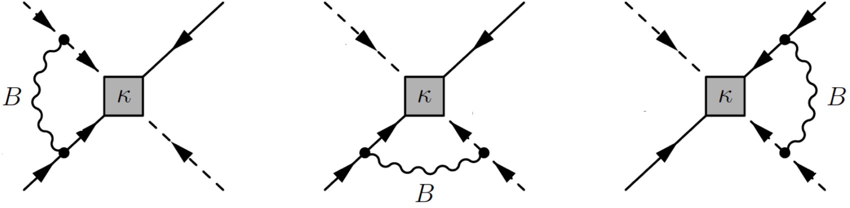
Relevant diagrams:



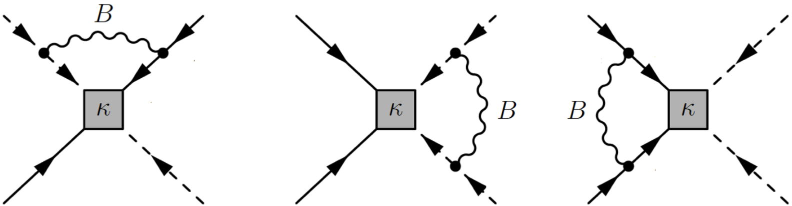
(a) $i\mu^\epsilon(\Gamma^\phi_\kappa)^{\beta\alpha}_{ij}$ (b) $i\mu^\epsilon(\Gamma^{e(1)}_\kappa)^{\beta\alpha}_{ij}$ (c) $i\mu^\epsilon(\Gamma^{e(2)}_\kappa)^{\beta\alpha}_{ij}$

(d) $i\mu^\epsilon(\Gamma^{e(3)}_\kappa)^{\beta\alpha}_{ij}$ (e) $i\mu^\epsilon(\Gamma^{e(4)}_\kappa)^{\beta\alpha}_{ij}$

Effective Vertex Renormalization



(a) $i\mu^{\epsilon}(\Gamma_{\kappa}^{B(1)})_{ij}^{\beta\alpha}$ (b) $i\mu^{\epsilon}(\Gamma_{\kappa}^{B(2)})_{ij}^{\beta\alpha}$ (c) $i\mu^{\epsilon}(\Gamma_{\kappa}^{B(3)})_{ij}^{\beta\alpha}$



(d) $i\mu^{\epsilon}(\Gamma_{\kappa}^{B(4)})_{ij}^{\beta\alpha}$ (e) $i\mu^{\epsilon}(\Gamma_{\kappa}^{B(5)})_{ij}^{\beta\alpha}$ (f) $i\mu^{\epsilon}(\Gamma_{\kappa}^{B(6)})_{ij}^{\beta\alpha}$

Similar contributions from the W bosons.

Effective Vertex Wavefunction Renormalization

Determine the counterterm:

$$(\Gamma_\kappa^{\phi})^{ij}_{\beta\alpha}\Big|_{\text{div}} + \sum_{s=1}^{4} (\Gamma_\kappa^{e(s)})^{ij}_{\beta\alpha}\Big|_{\text{div}} + \sum_{s=1}^{6} (\Gamma_\kappa^{B(s)})^{ij}_{\beta\alpha}\Big|_{\text{div}} + \sum_{s=1}^{6} (\Gamma_\kappa^{W(s)})^{ij}_{\beta\alpha}\Big|_{\text{div}}$$
$$+ \frac{1}{2}\Big[(\delta\kappa)^{ji}_{\beta\alpha}\varepsilon_{cd}\varepsilon_{ba} + (\delta\kappa)^{ij}_{\beta\alpha}\varepsilon_{ca}\varepsilon_{bd}\Big] P_L = 0$$

Effective Vertex Wavefunction Renormalization

Determine the counterterm:

$$(\Gamma_\kappa^{\phi})^{ij}_{\beta\alpha}\Big|_{\text{div}} + \sum_{s=1}^{4} (\Gamma_\kappa^{e(s)})^{ij}_{\beta\alpha}\Big|_{\text{div}} + \sum_{s=1}^{6} (\Gamma_\kappa^{B(s)})^{ij}_{\beta\alpha}\Big|_{\text{div}} + \sum_{s=1}^{6} (\Gamma_\kappa^{W(s)})^{ij}_{\beta\alpha}\Big|_{\text{div}}$$
$$+ \frac{1}{2}\Big[(\delta\kappa)^{ji}_{\beta\alpha}\varepsilon_{cd}\varepsilon_{ba} + (\delta\kappa)^{ij}_{\beta\alpha}\varepsilon_{ca}\varepsilon_{bd}\Big] P_L = 0$$

$$(\delta\kappa)^{ij}_{\beta\alpha} = -\frac{1}{16\pi^2}\Bigg\{ -\sum_{k,l=1}^{2} \kappa^{kl}_{\beta\alpha}\lambda_{kilj} + 2\sum_{k=1}^{2}\Big(\kappa^{kj}\, Y_e^{i\dagger}\, Y_e^{k} + \kappa^{jk}\, Y_e^{i\dagger}\, Y_e^{k} - \kappa^{ik}\, Y_e^{j\dagger}\, Y_e^{k}$$
$$+ Y_e^{kT}\, Y_e^{j*}\kappa^{ik} + Y_e^{kT}\, Y_e^{j*}\kappa^{ki} - Y_e^{kT}\, Y_e^{i*}\kappa^{kj}\Big)_{\beta\alpha}$$
$$+ \left(\xi_1 - \frac{3}{2}\right) g_1^2\kappa^{ij}_{\beta\alpha} + \left(3\xi_2 + \frac{3}{2}\right) g_2^2\kappa^{ij}_{\beta\alpha} - 3g_2^2\kappa^{ji}_{\beta\alpha}\Bigg\}\frac{1}{\epsilon}.$$

Effective Vertex β-Function

- The renormalization relation for the effective coupling $\kappa^{ij}_{\beta\alpha}$ is defined as

$$(\kappa_B)^{ij}_{\beta\alpha} = \left(Z_\phi^{-\frac{1}{2}}\right)^{ii'} \left(Z_{\ell_L}^{T-\frac{1}{2}}\right)_{\beta\beta'} \mu^\epsilon [\kappa + \delta\kappa]^{i'j'}_{\beta'\alpha'} \left(Z_{\ell_L}^{-\frac{1}{2}}\right)_{\alpha'\alpha} \left(Z_\phi^{-\frac{1}{2}}\right)^{j'j},$$

- The β-function for effective coupling is calculated to be

$$\begin{aligned} 16\pi^2 \mu \frac{\mathrm{d}\kappa^{ij}}{\mathrm{d}\mu} =& \Big\{ \kappa^{kl}\lambda_{kilj} + 2\Big[\kappa^{ik} Y_e^{j\dagger} Y_e^k - \left(\kappa^{kj} + \kappa^{jk}\right) Y_e^{i\dagger} Y_e^k \\ & + Y_e^{kT} Y_e^{i*} \kappa^{kj} - Y_e^{kT} Y_e^{j*} \left(\kappa^{ik} + \kappa^{ki}\right)\Big] + T^{ii'}\kappa^{i'j} + \kappa^{ij'} T^{j'j} \\ & + \frac{1}{2}\left(\kappa^{ij} Y_e^{k\dagger} Y_e^k + Y_e^{kT} Y_e^{k*} \kappa^{ij}\right) - 3g_2^2\left(2\kappa^{ij} - \kappa^{ji}\right) \Big\}. \end{aligned}$$

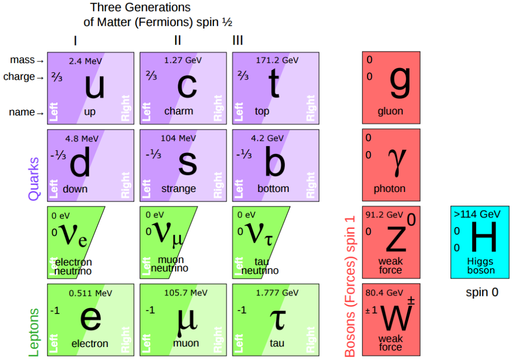
Outline

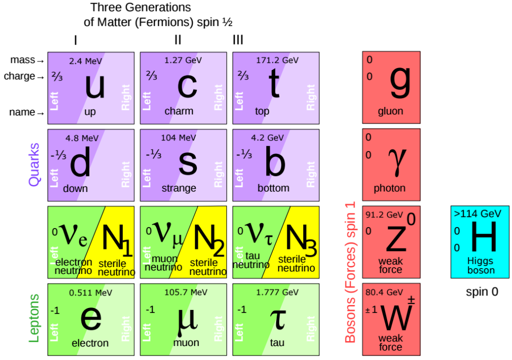
What's Next

- Extend the calculations above the 2HDM: first step - include RH neutrinos (N_R) and LH triplet scalar (Δ_L).
- Obtain other β-functions (e.g. for Yukawa couplings). Can be done by computer programs.
- Phenomenological study of neutrino mass in the LRSM.

Nature of Neutrinos

Minimal extension in the neutrino sector: right-handed/sterile neutrinos?





arXiv: 1301.5516 [hep-ph]