Chiral-Anomaly-Induced Nonlinear Hall Effect in Weyl Semimetals

Rui-Hao Li[1], Olle G. Heinonen[2], Anton A. Burkov[3], Steven S.-L. Zhang[1]

[1]Department of Physics, Case Western Reserve University, USA

[2]Materials Science Division, Argonne National Laboratory, USA

[3]Department of Physics and Astronomy, University of Waterloo, Canada

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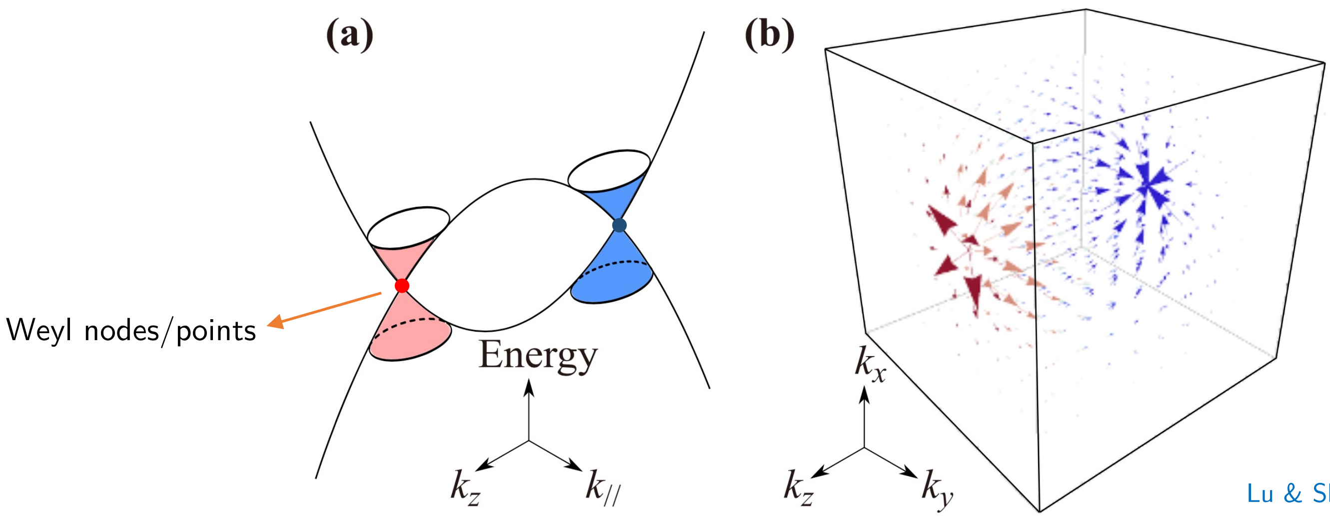
Weyl Semimetals

- Realization of Weyl fermions in condensed matter systems – Weyl Semimetals (WSMs).

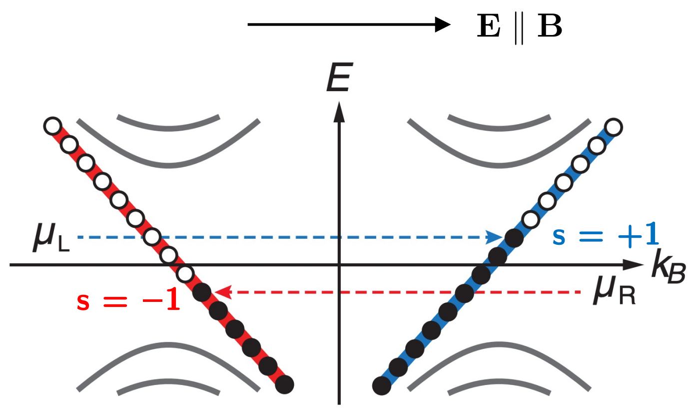


Lu & Shen, Front. Phys. (2017)

- Berry curvature: $\boldsymbol{\Omega}_n(\mathbf{k}) = \boldsymbol{\nabla}_{\mathbf{k}} \times \underbrace{\langle u_n(\mathbf{k})|i\boldsymbol{\nabla}_{\mathbf{k}}|u_n(\mathbf{k})\rangle}_{\text{Berry connection}}$

- Chern number: $C_n = \dfrac{1}{2\pi} \oint_{\mathrm{BZ}} \boldsymbol{\Omega}_n(\mathbf{k}) \cdot \mathrm{d}\mathbf{S}_{\mathbf{k}}$

Chiral Anomaly in WSMs

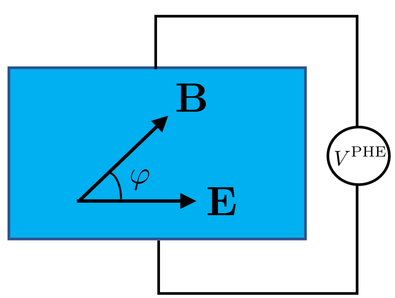


Kim, Ryoo & Park, PRL (2017)

- $\mu_L \neq \mu_R$
- $\dfrac{\mathrm{d}n_{R/L}}{\mathrm{d}t} = \pm\dfrac{e^2}{h^2}\mathbf{E}\cdot\mathbf{B}$

- Longitudinal negative magnetoresistance (NMR) [Nielson & Ninomiya, Phys. Lett. (1983); Son & Spivak, PRB (2013)]

- Planar Hall effect (PHE) [Burkov, PRB (2017); Nandy, Sharma, Taraphder & Tewari, PRL (2017)]



Nonlinear Response

- Both the NMR and PHE are linear responses to E field, i.e. $j \sim \mathcal{O}(E)$

- In this work, we proposed a nonlinear Hall effect induced by the chiral anomaly.

  - Chiral anomaly: $\delta n^s_{\mathbf{k}} \sim s(\mathbf{E} \cdot \mathbf{B})$ s: chirality

  - Anomalous velocity: $\mathbf{v}^s_a = \dfrac{e}{\hbar}\mathbf{E} \times \mathbf{\Omega}^s_{\mathbf{k}}$

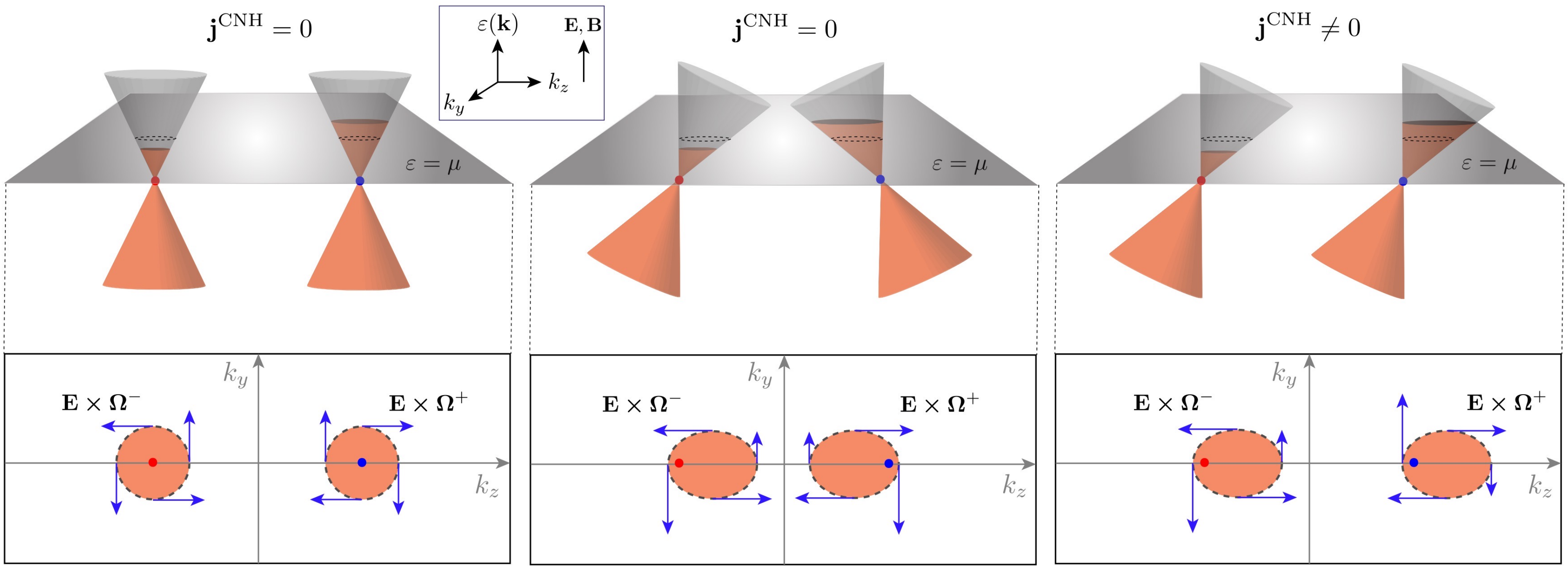
  - Chiral-anomaly-induced nonlinear Hall (CNH) current density:

$$\mathbf{j}^{\mathrm{CNH}} = -e \sum_{\mathbf{k},s} \delta n^s_{\mathbf{k}} \mathbf{v}^s_a \sim \mathcal{O}(E^2 B)$$

requires inversion-symmetry breaking

Schematics

Asymmetric Fermi surface ⇒ nonvanishing CNH effect



(a) (b) (c)

Tilted WSMs

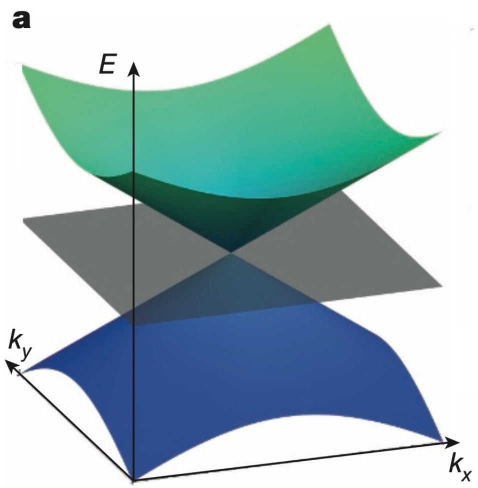
$$\mathcal{H}^s(\mathbf{k}) = \hbar v_F (s\mathbf{k}\cdot\boldsymbol{\sigma} + R_s k_z \sigma_0)$$

$$\varepsilon^s(\mathbf{k}) = \hbar v_F (R_s k_z \pm k)$$

+: conduction band
−: valence band

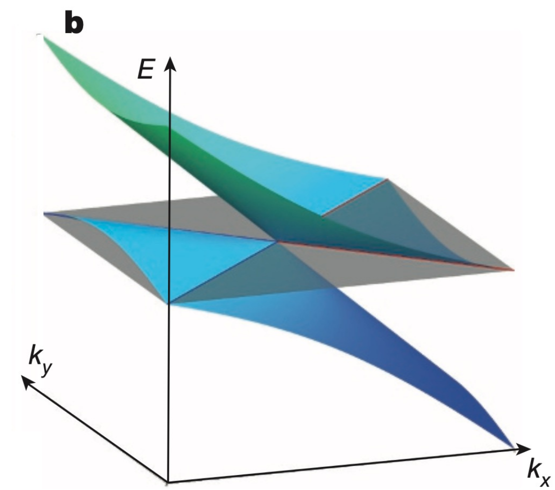
$$\boldsymbol{\Omega}^s(\mathbf{k}) = -s\frac{\pm\mathbf{k}}{2k^3}$$

Type-I



$|R_s| < 1$

Type-II



$|R_s| > 1$

Soluyanov et al., Nature (2015)

Semiclassical Boltzmann Equations

- EoM of electrons in a WSM:

$$D^s \dot{\mathbf{r}}^s = \mathbf{v}^s + \frac{e}{\hbar}\mathbf{E} \times \mathbf{\Omega}^s + \frac{e}{\hbar}(\mathbf{v}^s \cdot \mathbf{\Omega}^s)\mathbf{B},$$

$$D^s \dot{\mathbf{k}}^s = -\frac{e}{\hbar}\mathbf{E} - \frac{e}{\hbar}\mathbf{v}^s \times \mathbf{B} - \frac{e^2}{\hbar^2}(\mathbf{E} \cdot \mathbf{B})\mathbf{\Omega}^s.$$

Modified density of states:

$$D^s \equiv 1 + \frac{e}{\hbar}(\mathbf{B} \cdot \mathbf{\Omega}^s)$$

- Boltzmann equation with relaxation time approximation:

$$\underbrace{\cancel{\frac{\partial f^s}{\partial t}}}_{\text{steady-state}} + \underbrace{\cancel{\dot{\mathbf{r}}^s \cdot \frac{\partial f^s}{\partial \mathbf{r}^s}}}_{\text{homogeneous}} + \dot{\mathbf{k}}^s \cdot \frac{\partial f^s}{\partial \mathbf{k}^s} = -\frac{f^s - f_0^s}{\tau}$$

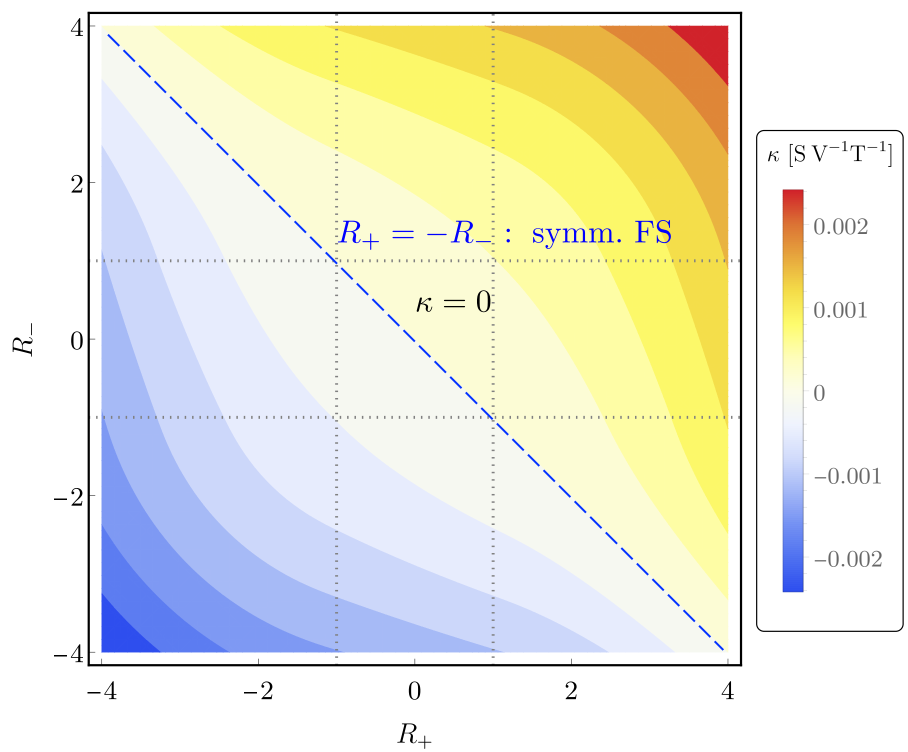
- Current density:

$$\mathbf{j} = (-e)\sum_s \int_{\mathbf{k}} D^s\, \dot{\mathbf{r}}^s\, f^s$$

Nonlinear Response Functions

- Chiral-anomaly-induced nonlinear Hall (CNH) current density:

$$\boxed{\mathbf{j}^{\mathrm{CNH}} = \sum_s \kappa^s (\mathbf{E}\cdot\mathbf{B})(\mathbf{E}\times\hat{\mathbf{t}})}$$



$$\kappa = \sum_s \kappa^s = \kappa^+ + \kappa^-$$

Comparison to other nonlinear Hall effects

- CNH in component form:

$$j_a^{\mathrm{CNH}} = \sum_s \varkappa_{abcd}^s E_b E_c B_d$$

$$\varkappa_{abcd}^s = \epsilon_{abl}\epsilon_{gcm}\epsilon_{gdn}\frac{e^4\tau}{\hbar^3}\int_{\mathbf{k}} f_0^s \frac{\partial}{\partial k_n}\left(\Omega_l^s \Omega_m^s\right)$$

- NHE induced by the Berry curvature dipole in systems with TRS:

[Sodemann & Fu, PRL (2015); Low, Jiang & Guinea, PRB (2015)]

$$j_a^{\mathrm{BNH}} = \sum_s \chi_{abc}^s E_b E_c$$

$$\chi_{abc}^s = \epsilon_{acd}\frac{e^3\tau}{2}\underbrace{\int_{\mathbf{k}} f_0^s \frac{\partial}{\partial k_b}\Omega_d^s}_{\text{Berry curvature dipole}}$$

- NHE induced by disorder: extrinsic, side-jump + skew-scattering

[Du, Wang, Li, Lu & Xie, Nat. Commun. (2019); Xiao, Du & Niu, PRB (2019)]

Summary

- We predicted a nonlinear Hall effect in tilted Weyl semimetals (WSMs), which arises from the concerted actions of the chiral anomaly and the anomalous velocity:

$$\mathbf{j}^{\mathrm{CNH}} = \sum_s \kappa^s (\mathbf{E} \cdot \mathbf{B})(\mathbf{E} \times \hat{\mathbf{t}})$$

- This effect is inherently different from the nonlinear Hall effect originated from the Berry curvature dipoles [see e.g., Sodemann & Fu, PRL (2015)].

- An asymmetric Fermi surface is crucial to attaining a non-vanishing chiral-anomaly-induced nonlinear Hall current in noncentrosymmetric WSMs.

- CNH effect can be detected with second-harmonic generation measurements in the a.c. regime, or through proper alignments of E and B fields in the d.c. regime.

Back-up Slides

CNH Current Density & Fermi Surface

- Why is an asymmetric Fermi surface necessary?

$$\mathbf{j}^{\mathrm{CNH}} = \frac{e^4\tau}{\hbar^2}\sum_s \int_{\mathbf{k}} \frac{\partial f_0^s}{\partial \varepsilon^s} \mathbf{E}\times\boldsymbol{\Omega}^s(\mathbf{E}\times\boldsymbol{\Omega}^s)\cdot(\mathbf{v}^s\times\mathbf{B})$$

- Untilted Weyl cones:

$$\boldsymbol{\Omega}^s(-\mathbf{k}) = -\boldsymbol{\Omega}^s(\mathbf{k})$$

$$\varepsilon^s(-\mathbf{k}) = \varepsilon^s(\mathbf{k}) \;\Rightarrow\; \mathbf{v}^s(-\mathbf{k}) = -\mathbf{v}^s(\mathbf{k})$$

- Two oppositely tilted Weyl cones:

$$\boldsymbol{\Omega}^s(\mathbf{k}) = \boldsymbol{\Omega}^{-s}(-\mathbf{k})$$

$$\varepsilon^s(\mathbf{k}) = \varepsilon^{-s}(-\mathbf{k}) \;\Rightarrow\; \mathbf{v}^s(\mathbf{k}) = -\mathbf{v}^{-s}(-\mathbf{k})$$

$$\mathbf{j}^{\mathrm{CNH}} = 0$$

Inversion Breaking $\not\Rightarrow$ Asymmetric FS

- E.g. TI-NI multilayer with broken inversion symmetry [Zyuzin, Wu & Burkov, PRB (2012)]

$$\mathcal{H}(\mathbf{k}) = v_F\tau^z(\hat{\mathbf{z}}\times\boldsymbol{\sigma})\cdot\mathbf{k} + \Delta_S\tau^x + \frac{1}{2}\big(\Delta_D\tau^+ e^{ik_zd} + \text{h.c.}\big) + \underbrace{V\tau^z + \lambda\tau^y\sigma^z}_{\text{Inversion breaking}}$$